Binomial Theorem

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Ms. Khushi and Mr. Daksh decide to construct a Pascal triangle with the help of binomial theorem. They use the formula for the expansion is

$$(x+y)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} y^r$$
$$= {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + \ldots + {}^nC_{n-1} x^1 y^{n-1} + {}^nC_n x^0 y^n.$$

(A) The coefficient of x^k ($0 \le k \le n$) in the expansion of E = 1 + (1 + x) + $(1 + x)^2$ +... (1 + x)n is:

(a) ${}^{n+1}C_{k+1}$

(b) ${}^nC_k$

(c) ${}^{n+1}C_{n-k-1}$

(d) none of these

(B) The coefficient of y is the expansion of

$$\left(y^2 + \frac{c}{y}\right)^5 \text{ is:}$$

(a) 10 c^3

(b) 20 c^2

(c) 10 c

(d) 20 c

(C) The number of terms in the expansion of $(1+ \sqrt{5}x)^2 + (1 - \sqrt{5}x)'$ are:

(a) 4

(b) 8

(c) 5

(d) 9

(D) The sum of coefficient of even powers x in

the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is:

(a) $11 \times {}^{11}C_5$ (b) $\frac{11}{2} \times {}^{11}C_6$

(c) $11({}^{11}C_5 + {}^{11}C_6)$ (d) 0

(E) Assertion (A): The value of (101)* using the binomial theorem is 104060401.

Reason (R): $(x+y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}.y + {}^nC_2x^{n-2}.y^2 + \ldots + {}^nC_ny^n$.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Ans. (A) (a) ${}^{n+1}C_{k+1}$

Explanation:

$$E = \frac{(1+x)^{n+1}-1}{(1+x)-1}$$

$$= \frac{{}^{n+1}C_0 + {}^{n+1}C_1x + {}^{n+1}C_2x^2 + \ldots - 1}{x}$$

$$= {}^{n+1}C_1 + {}^{n+1}C_2x + {}^{n+1}C_3x^2 + \ldots$$

Coefficient of $x^4 = {}^{n+1}C_{k+1}$

(B) (a) $10\ c^3$

Explanation:

$$\left(y^2+\frac{c}{y}\right)^5 = {}^5C_0\left(\frac{c}{y}\right)^0(y^2)^{5-0} + {}^5C_1\left(\frac{c}{y}\right)^1(y^2)^{5-1} + \ldots + {}^5C_5\left(\frac{c}{y}\right)^5(y^2)^{5-5}$$

$$= \sum_{r=0}^{5} {}^5C_r\left(\frac{c}{y}\right)^r(y^2)^{5-r}$$

We need coeffcient of y

$\Rightarrow \quad 2(5-r)-r=1$

$\Rightarrow \quad 10-3r=1$

$\Rightarrow \quad r=3$

So, coefficient of $y = {}^5C_3 . c^3$

$= 10c^3$

(C) (a) 4

Explanation: Given expansion is

$(1+\sqrt{5}x)^7 + (1-\sqrt{5}x)^7$

Here, n = 7, which is odd.

$$\text{Total number of terms} = \frac{n+1}{2}$$

$$= \frac{7+1}{2}$$

$$= \frac{8}{2}$$

$= 4$

(D) (d) 0

Explanation: (r+1)th term = $^{11}Cr\ (x)^{11}\text{-}rx^{-r}$

$= {}^{11}C_r.x^{11-2r}$

Even power of x exists only if 11 - 2r = an

even number which is not possible Thus, Sum of coefficient = 0

(E) (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: Given: $(101)^4$

Here, 101 can be written as the sum or the

difference of two numbers, such that the

binomial theorem can be applied.

Therefore, 101 = 100 + 1

Hence, $(101)^4\ (100 + 1)^4$

Now, by applying the binomial theorem, we get

$(101)^4 = (100+ 1)^4 = {}^4C_0(100)^4$

$+{}^4C_1\ (100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)\ (1)^3 + {}^4C_4(1)^4$

$(101)^4 = (100)4 + 4\ (100)3 + 6(100)2 + {}^4(100) + (1)4$

(101) = 100000000 + 4000000 + 60000 + 400 + 1

$(101)^4$ 104060401

2. Four friends applied the knowledge of Binomial Theorem while playing a game to make the equations by observing some conditions they make some equations.

(A) Expand, $(1-x+x^2)^4$.

(B) Expand the expression, $(1 - 3x)^7$

(C) Show that $11^9 + 9^{11}$ is divisible by 10.

Ans. (A) We have,

$(1-x+x^2)^4= [(1-x)+x^2]^4$

$= {}^4C_0(1 - x)^4 + {}^4C_1(1 - x)^3 (x^3) + {}^4C_2(1 - x)^2$

$(x^2)^2+{}^4C_3(1-x) (x^2)_3 + {}^4C_4(x^2)^4$

$= (1-x)^4 + 4x^2 (1-x)^3 + 6x^4(1-x)^2+4x^6 (1-x) + 1.x^8$

$= (1-4x+6x^2 - 4x^3 + x^4) + 4x^2 (1 - 3x + 3x^2 -x^3)+6x^4+ (1 - 2x + x^2)+4(1-x)x^6 + x^8$

$= 1-4x+6x^2 - 4x^3 + x^2 + 4x^2 - 12x^3 + 12x^4$

$- 4x^5+6x^4-12x^5 + 6x^6 + 4x^6 - 4x^7+x^8$

$= 1-4x+10x^2-16x^3+ 19x^4 - 16x^5 + 10x^6$

$- 4x^7+x^8$

(B) Here, a = 1, b = 3x, and n = 7

Given, $(1-3x)^7={}^7C_0(1)^7 -7c_1(1)^6 (3x)^1 +{}^7C_2(1)^5 (3x)^2-{}^7C_3(1)^1 (3x)^3 +{}^7C_4(1)^3 (3x)^4$

$-{}^7C_5(1)^2 (3x)^5+{}^7C_6(1)^1 (3x)^6$

$-{}^7C_7(1)^0 (3x)^7$

$= 1-21x+189x^2-945x^3 + 2835x^4-5103x^5+5103x^6-2187x^7$.

(C)

$11^9 + 9^{11} = (10 + 1)^9 + (10 - 1)^{11}$

$= ({}^9C_0 . 10^9 + {}^9C_1.10^8 + ... {}^9C_9)$

$+ ({}^{11}C_0.10^{11} - {}^{11}C_1.10^{10} + ... - {}^{11}C_{11})$

$= {}^9C_0.10^9 + {}^9C_1.10^8 + ... + {}^9C_8.10 + 1 + 10^{11}$

$- {}^{11}C_1.10^{10} + ... + {}^{11}C_{10}.10 - 1$

$= 10[{}^9C_0.10^8 + {}^9C_1.10^7 + ... + {}^9C_8$

$+ {}^{11}C_0.10^{10} - {}^{11}C_1.10^9 + ... + {}^{11}C_{10}]$

= 10 K, which is divisible by 10.